

# Working with Square Roots

Tanya Khovanova

December 14, 2009

Who is inclined towards mathematics? Take the shovels and extract roots.

## Class Discussion

How to work with expressions, where  $p_i$  are polynomials in  $x$ . Cases:  $p_1 + \sqrt{p_2} = p_3$ ,  $\sqrt{p_1} + \sqrt{p_2} = p_3$ ,  $\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3} = p_4$ ,  $\sqrt{p_1} + \sqrt{p_2} + \sqrt{p_3} + \sqrt{p_4} = 0$ . Rejoice when you have:  $\sqrt{p_1 + \sqrt{p_2}} + \sqrt{p_1 - \sqrt{p_2}}$ .

## Warm Up

**Exercise 1.** Genetically engineered octopuses with an even number of arms always tell the truth, the ones with an odd number of arms always lie. Only octopuses with six, seven or eight arms are allowed to serve Neptune. The four guards from the last shift of guards at the palace has nothing better to do than count their arms: The pink one said, “Gray and I have 15 arms together.” The gray one said, “Lavender and I have 14 arms together.” The lavender one said, “Turquoise and I have 14 arms together.” The turquoise one said, “Pink and I have 15 arms together.”

What number of arms does each one have?

**Exercise 2.** Write a quadratic equation one root of which is the sum and the other is the product of roots of  $ax^2 + bx + c = 0$ .

## Problem Set

**Exercise 3.** Factor:  $x^4 + 2x^3 + 4x^2 + 3x + 2$ .

**Exercise 4.** Simplify:

$$\left(\frac{1 + \sqrt{x}}{\sqrt{1+x}} - \frac{\sqrt{1+x}}{1 + \sqrt{x}}\right)^2 - \left(\frac{1 - \sqrt{x}}{\sqrt{1+x}} - \frac{\sqrt{1+x}}{1 - \sqrt{x}}\right)^2.$$

**Exercise 5.** Prove that:

$$(4 + \sqrt{15})(\sqrt{10} - \sqrt{6})\sqrt{4 - \sqrt{15}} = 2.$$

**Exercise 6.** Solve:  $1 + \sqrt{1 + x\sqrt{x^2 - 24}} = x$ .

**Exercise 7.** Solve:  $\sqrt{3x + 4} + \sqrt{x - 4} = 2\sqrt{x}$ .

**Exercise 8.** Solve:  $\sqrt{x + \sqrt{x + 11}} + \sqrt{x - \sqrt{x + 11}} = 4$ .

**Exercise 9.** Find  $z^5 + 1/z^5$ , given that  $z > 0$  and  $z^2 + 1/z^2 = 14$ .

**Exercise 10.** Write a quadratic equation such that its roots are equal to the roots of  $ax^2 + bx + c = 0$  plus 1.

**Exercise 11.** Find an equation with integer coefficients that has  $\sqrt{2} + \sqrt{3}$  as its root.