

# Rotations 2

Tanya Khovanova

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Without geometry life would be pointless.

## Warm Up

**Exercise 1.** A certain sheik named Hassan has eight horses. Four of them are white, three are black, and one is brown. Assuming now that Hassan's horses can talk and always tell the truth, how many of them will each say that it is the same color as another one of Hassan's horses?

## Competition Practice

**Exercise 2. AMC 8 1999.** In trapezoid  $ABCD$ , the side  $AB$  and  $CD$  are equal. The parallel sides  $AD$  and  $BC$  are 8 and 16. The height between them is 3. What is the perimeter of  $ABCD$ ?

**Exercise 3. AIME 1983.** The length of diameter  $AB$  is a two-digit integer. Reversing the digits gives the length of a perpendicular chord  $CD$ . The distance from their intersection point  $H$  to the center  $O$  is a positive rational number. Determine the length of  $AB$ .

**Exercise 4. AIME 1984.** A point  $P$  is chosen in the interior of  $\triangle ABC$  such that when lines are drawn through  $P$  parallel to the sides of  $\triangle ABC$ , the resulting smaller triangles  $t_1$ ,  $t_2$ , and  $t_3$  have areas 4, 9, and 49, respectively. Find the area of  $\triangle ABC$ .

**Exercise 5. AIME 1984.** Find the value of  $a_2 + a_4 + a_6 + a_8 + \dots + a_{98}$  if  $a_1, a_2, a_3, \dots$  is an arithmetic progression with common difference 1, and  $a_1 + a_2 + a_3 + \dots + a_{98} = 137$ .

**Exercise 6. HMNT 2009. Guts Round.** The angles of a convex  $n$ -sided polygon form an arithmetic progression whose common difference (in degrees) is a non-zero integer. Find the largest possible value of  $n$  for which this is possible. (A polygon is convex if its interior angles are all less than  $180^\circ$ .)

**Exercise 7. Spivak middle school Olympiad.** Ivan met two two-headed dragons, whose heads were tangled. Dragons could be of two types: truth-tellers and liars. All the heads of the same dragon behave the same: either tell the truth or lie. Ivan decided to help the dragons to untangle their heads, for this it would have been useful to know which head is which. He asked that every head and here are the answers:

- First head: “I am a truth-telling head”.
- Second head: “The third head is my joint head”.
- Third head: “The second head is not my joint head”.
- Fourth head: “The third head is lying”.

Which head is joint with the first head?

## Challenge Problems

**Exercise 8.** Prove that a composition of a translation and a rotation is a rotation.

**Exercise 9.** There are 2009 towns in Russia and all have airports. All flights are heavily regulated by Russian government. Every day there are the same flights and all flights depart at noon, so you can only take one flight per day. Eventually you can get from any town to any other town, maybe with some connections. James Bond is trying to catch Dr. No. He can get him for sure if they arrive at the same town on the same day. Unfortunately for Dr. No, Bond has tagged him with a tracking device, so Bond always knows where Dr. No is. Dr. No never flies on Sundays (but James Bond can). Assuming that Dr. No is stuck in Russia, prove that James Bond can catch him sooner or later.

**Exercise 10.** Is it possible to place 4 players on a football field so that the 6 distances between them would equal 1, 2, 3, 4, 5, and 6 yards?