Reflections

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Class Discussion

M.C. Escher. Reflections.

We will start by discussing *line symmetry*. For this we need to have a line, which we will denote as L. This line is also called the axis of symmetry. Line symmetry is so important that it has many different names: *reflection symmetry*, *mirror-symmetry*, *mirror-image symmetry* or *bilateral symmetry*.

A point A' is said to be the reflection of point A with respect to line L, if the segment AA' is perpendicular to L and is bisected by L. We also say that A' is symmetric to A with respect to line L. The first statement: If A' is the reflection of A, then A is the reflection of A'.

The reflection of a figure F is the figure that is formed from the points that are the reflections of all the points in F.

We see that reflection preserves the shapes of objects. So far, we have discussed only symmetrical objects. If you reflect a non-symmetrical object, the orientation of this object will change.

We say that a figure F has a line of symmetry if there is a line L such that the figure F is symmetrical to itself with respect to this line.

Warm Up

Exercise 1. "I guarantee", said the pet-shop salesman, "that this parrot will repeat every word it hears." A customer bought the parrot but found it wouldn't speak a single word. Nevertheless, the salesman told the truth. Explain.

Exercise 2. A solid, four-inch cube of wood is coated with blue paint on all six sides. Then the cube is cut into smaller one-inch cubes. These new one-inch cubes will have either three blue sides, two blue sides, one blue side, or no blue sides. How many of each will there be?

Problem Set

Exercise 3. Let AB be a segment, and let A' and B' be the reflections of points A and B with respect to line L. Prove that the segment $A'B'$ is the reflection of the segment AB.

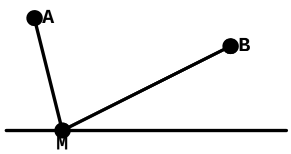
Exercise 4. Prove that a reflection of a circle is a circle.

Exercise 5. For the capital letters of the English alphabet find the letters that have a line of symmetry. Are there any letters that have more than one line of symmetry? (Strictly speaking, the symmetry of the letters depends on the font. But I can imagine a font in which the letter H has two lines of symmetry.)

Exercise 6. Prove that if a figure has exactly two intersecting lines of symmetry, then these lines are perpendicular.

Exercise 7. Can a figure have an infinite number of lines of symmetry? Can a bounded figure have an infinite number of lines of symmetry?

Exercise 8. You have line L and points A and B on the same side. Find a point M on line L such that the sum of distances $AM + MB$ is minimal.



Exercise 9. There is an infinite wall on the plane in the form of a straight line. You have the materials to build an extra piece of wall of any shape of length M. For some strange reason you want to build an enclosure of the maximal area and you can use the existing piece of wall. What shape should your wall be?