Invariants

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Assume, for the sake of clarity, that that yellow cube is a blue sphere.

Class Discussion

Prediction trick. http://quirkology.com/USA/Video_PredictionTrick.shtml

Warm Up

Exercise 1. How do we know that the total number of mahjong pieces is even?

Exercise 2. Martians have two letters in their alphabet: q and j. Their words have no more than 10 letters. If you remove a substring qj or jq from a word, its meaning doesn't change. How many different meanings can they have?

Exercise 3. The magic apple tree on Mars can grow bananas and oranges. Each tourist approaching the tree is allowed to pick exactly two fruits. If s/he picks two bananas or two oranges, the tree immediately grows one more orange; if s/he picks one banana and one orange the tree grows one more banana. Yesterday it had 100 bananas and 100 oranges. Today the group of AMSA students had a tour, and only one fruit is left. Which is it?

Exercise 4. On a chessboard, two cells located in opposite corners are cut out. Can we cover the rest of the board with dominoes (that is, 1×2 rectangles)?

Exercise 5. A coin changing machine changes one dollar coin into 4 quarters and one quarter into a dime and three nickels. Is it possible to change 10 dollar coins into exactly 24 coins?

Exercise 6. The integers from 1 to 2005 are written on a blackboard. You are allowed to erase any two numbers replacing them with one number that is equal to their difference. Can you make all the numbers on the blackboard be zero?

Exercise 7. Reversi buttons are placed on a chess board. Each of the 64 cells has a button. One of the buttons is black, and all the others are white. In one move you are allowed to flip all the buttons in one column or in one row. Can you make all the buttons white? What if the chessboard is 5 by 5?

Challenge Problems

Exercise 8. The integers from 1 to 2005 are written on a blackboard. You are allowed to erase any two numbers a and b replacing them with $a+b-1$. At the end there is one number left on the board. What can it be?

Exercise 9. Martians can be of three different colors: red, blue and purple. Today 13 red, 15 blue and 17 purple Martians arrived to visit AMSA. Martians love shaking hands with each other. Each time two different colored Martians shake their hands they change color to the third color. Is it possible that after the tour of AMSA, all of the Martians will be the same color?

Exercise 10. We put seven zeroes and one one in the vertices of a cube. In one move you are allowed to add 1 to the two numbers at the endpoints of some edge of the cube. Can you make all the numbers the same after several moves? Can you make all the numbers divisible by 3?

Exercise 11. You start with the number $2008^{2009}+2009^{2008}$ and then replace it with the sum of its digits. Then you calculate the sum of the digits of the result and you repeat the procedure until you get a one-digit number. Assuming that you are doing your calculations correctly, what is you final answer?

Exercise 12. 44 trees arranged themselves in a circle. There is a bird on each tree. From time to time two birds change their trees. One of the birds flies to the next tree clockwise and the other to the next tree counterclockwise. Can all the birds end up on the same tree?