Calendar and Coins Review

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Class Discussion

Sara's Birthday: http://blog.tanyakhovanova.com/?p=216.

Warm-up

Exercise 1. Alex wants to celebrate his birthday at the moment the Earth passes the same point in its orbit when he was born. He was born on September 22, 1991 at 1 am. Assuming that a year is 365 days and 6 hours, create his celebration schedule.

Exercise 2. What is the largest amount of money one can have in coins without being able to make change for a dollar?

Exercise 3. In how many ways can you have 25 U.S. coins whose total is $1.00?

Exercise 4. I have an equal number of pennies, nickels and dimes. I also have some quarters which have the same value as the pennies, nickels and dimes combined. If I have no other coins, what is the fewest possible total number of coins I could have? What is the value of all the coins?

Exercise 5. You have ten boxes; each contains nine balls. The balls in one box weigh 0.9 kg; the rest weigh 1.0 kg. You have one weighing on an accurate scale to find the box containing the light balls. How do you do it?

Competition Practice

Exercise 6. HMMT 1999. For what single digit n does 91 divide the 9-digit number $12345n789$?

Exercise 7. HMMT 1999. How many ways are there to cover a 3×8 rectangle with 12 identical dominoes?

Challenge Problems

Exercise 8. Eight coins weighing $1, 2, \ldots, 8$ grams are given, but which weighs how much is unknown. Baron Münchhausen claims he knows which coin is which; and offers to prove himself right by conducting one weighing on a balance scale, so as to unequivocally demonstrate the weight of at least one of the coins. Is this possible, or is he exaggerating?

Exercise 9. You have 6 coins weighing 1, 2, 3, 4, 5 and 6 grams that look the same, except for their labels. The number (1, 2, 3, 4, 5, 6) on the top of each coin should correspond to its weight. How can you determine whether all the numbers are correct, using the balance scale only twice?

Exercise 10. You have a balance scale and 13 coins, 1 of which is counterfeit. The counterfeit coin weighs less or more than the other coins. Can you determine the counterfeit in 3 weightings? You do not have to tell if it is heavier or lighter.

Exercise 11. You have a balance scale and 12 coins, 1 of which is counterfeit. The counterfeit coin weighs less or more than the other coins. Can you determine the counterfeit in 3 weightings and tell if it is heavier or lighter. In this more difficult version you have to discribe your three weighings in advance. That is, the next weighing can not depend on the previous weighing.