Set Theory

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Class Discussion

Set Theory. Venn Diagrams. Inclusion-Exclusion Principle.

Warm Up

Exercise 1. Decode:

$$OXOXO + AXAXA = AXAXAX.$$

Every letter represents a different digit.

Exercise 2. 10% of all mathematicians also do philosophy. 1% of philosophers do mathematics. Are there more philosophers or mathematicians?

Problem Set

Exercise 3. If January has 4 Mondays and 4 Fridays, what day of the week is January 20?

Exercise 4. Once Tanya made a very difficult quiz. Those students who failed came to the next make-up round of the quiz. Each round, one third of the students plus one third of a student passed the quiz. What is the minimal possible number of students who failed all of the rounds, if there were 5 rounds?

Exercise 5. Tanya has 35 students at the math club. 20 of them like singing, 11 of them like dancing, and 10 of them like neither singing nor dancing. How many of Tanya's students like both singing and dancing?

Exercise 6. Tanya has 100 students at the math club: 85 of them speak Russian, 80 Spanish, 75 Latin. How many of Tanya's students are guaranteed to know all three languages?

Exercise 7. Tanya is teaching her students to count. As a challenge problem, Tanya asked her students to count their pens, pencils and markers. Out of 40 students ten have the same number of pens as pencils, and 15 have a different number of pens and markers. Prove that there are at least 15 students who have a different number of pencils and markers.

Exercise 8. Among the numbers from 1 to 2008 inclusive are there more numbers divisible by 9 and not divisible by 8, or numbers divisible by 8 and not divisible by 9?

Exercise 9. How many numbers between 1 and 1000 are not divisible by 3 or 5? The same question about 2, 3 or 5?

Exercise 10. Calculate:

$$\left(\frac{(2.7 - 0.8) \cdot 2\frac{1}{3}}{(5.2 - 1.4)/\frac{3}{70}} + 0.125 \right) /2\frac{1}{2} + 0.43.$$

Exercise 11. Mike and Bob stepped on the escalator going down. While Mike was walking down he stepped on 50 steps. Bob was walking 3 times faster and stepped on 75 steps. Assuming they were not skipping steps, how many steps would they have needed to step if the escalator weren't moving?

Exercise 12. When Nick was as old as Ann is now, their aunt, Petunia, was one year less than Nick and Ann together now. How old was Nick when Petunia was as old as Nick is now?