Sequences

Tanya Khovanova

January 26, 2009

How to prove that all odd numbers are prime ?
Physicist: 3 is prime, 5 is prime, 7 is prime, 9 is an experimental error...
Quantum Physicist: All numbers are equally prime and non-prime until observed.
Professor: 3 is prime, 5 is prime, 7 is prime, and the rest are left as an exercise for the student.
Measure nontheorist: There are exactly as many odd numbers as primes, and exactly one even prime (namely 2), so there must be exactly one odd nonprime (namely 1).
Computer Scientist: 10 is prime, 11 is prime, 101 is prime...
Programmer: 3 is prime, 5 is prime, 7 is prime, 9 will be fixed in the next release, ...
Windows programmer: 3 is prime. Wait...
Computer programmer: 3 is prime, 5 is prime, 7 is prime, 7 is prime, 7 is prime, 7 is prime, 7 is ...
Computational linguist: 3 is an odd prime, 5 is an odd prime, 7 is an odd prime, 9 is a very odd prime, ...
Philosopher: Why don't we just call all the odd numbers prime and call all the prime numbers odd, that way all the odd numbers would be prime.
Statistician: 100% of the sample 5, 13, 37, 41 and 53 is prime, so all odd numbers must be prime.

Class Discussion

Sequences. Continuing sequences. Complexity of a sequence.

Warm Up

Exercise 1. The day before yesterday I was 25 and the next year I will be 28. (This is true only one day in a year.) When was I born?

Exercise 2. What mathematical symbol can be put between 5 and 9, to get a number bigger than 5 and smaller than 9?

Problem Set

Exercise 3. What is the next term of the following sequences:

1. 1, 4, 9, 16, 25,
2. 1, 3, 6, 10, 15, 21,
3. 1, 1, 2, 3, 5, 8, 13,
4. 2, 7, 1, 8, 2, 8, 1, 8, 2,
5. 1, 2, 3, 2, 1, 2, 3, 4, 2, 1, 2,
6. 1, 1, 2, 1, 2, 2, 3, 1, 2, 2, 3, 2, 3, 3, 4, 1,
7. 3, 3, 5, 4, 4, 3, 5, 5, 4, 3,
8. 1, 3, 4, 7, 11, 18, 29, 47,
9. 1, 11, 21, 1211, 111221, 312211, 13112221,
10. 3, 6, 11, 18, 27, 38, 51,
11. 1, 2, 6, 20, 70, 252, 924, 3432, 12870, 48620,
12. 1, 2, 2, 3, 3, 4, 4, 4, 5, 5, 5, 6, 6, 6, 6, 7, 7, 7, 7, 8, 8, 8, 8, 9, 9, 9, 9,

Exercise 4. Prove that:

$$\binom{n}{0} - \binom{n}{1} + \binom{n}{2} - \binom{n}{3} + \ldots + (-1)^{n-1}\binom{n}{n-1} + (-1)^n\binom{n}{n} = 0.$$