

Numbers

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May 8, 2008

Announcement

Tanya is going to a conference in Princeton. The next lesson will be on May 29, 2008.

Warm Up

Exercise 1. We have 10 fingers on our hands. How many fingers are there on 10 hands?

Exercise 2. Three AMSA students were helping Tanya make Möbius strips for the next lesson. They made three strips in three minutes. How many strips can 12 AMSA students make for Tanya in 12 minutes?

Problem Set

Exercise 3. How much bigger is the sum of all even numbers up to 100 than the sum of all odd numbers up to 100?

Exercise 4. What digits should you replace the stars with in the number 517^{**} so that the result is divisible by 6, 7 and 9?

Exercise 5. Find the smallest positive integer x such that half of x is a square, a third of x is a cube, and a fifth of x is a fifth power.

Exercise 6. How many numbers below 1000 are not divisible by 3 or 5? Another question: how many numbers are not divisible by 2 or 3 or 5?

Exercise 7. A number ends with the digit 2. If we move this digit to the front, the resulting number will be twice as big as the initial number. Find the smallest possible initial number.

Exercise 8. How many different necklaces can you build from 5 blue stones and 2 red stones?

Exercise 9. I have 5 different brainteaser puzzles and 10 students in my training class. In how many different ways I can give out the brainteasers to the students? Solve for two different cases: I can give only one brainteaser per student, or I can give several brainteasers to a student.

Exercise 10. You have a balance scale and 12 coins, one of which is counterfeit. The counterfeit weighs less or more than the other coins. Can you determine the counterfeit in 4 weightings, and tell if it is heavier or lighter? What about 3 weightings?

Exercise 11. Find the sum of all four-digit even numbers which you can write using the digits 0, 1, 2, 3, 4, 5 (you are allowed to reuse digits).

Exercise 12. Numbers are written one after another: 1234567891011... Find the digit at the 206788-th place.

Exercise 13. Prove that the sum of five consecutive squares is never a square.